A New Model for Microwave Characterization of Composite Materials in Guided-Wave Medium

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***Abstract*— A method of determining the permittivity and the permeability of heterogeneous materials from microwave measurements in a coaxial line or in a rectangular waveguide is presented. Fluctuations are observed in the curves of the transmission and reflection coefficients measured in a guided space cell which are caused by the propagation of modes higher than lowest order. The measuring cell containing the sample is represented by an unperturbed line in series with resonators which model the coupling between the sample and the measurement cell for each higher mode resonating inside the sample. Finally, the intrinsic characteristics of the material are computed from the data for the unperturbed line. Results for several composite materials and measurement cells are presented to demonstrate the capabilities of this model.**

I. Introduction

COMPOSITE materials are of great interest in microwave applications. In this context, we need to determine their permittivity ϵ and permeability μ versus frequency.

At present, classic methods (T/R in lines or free space, perturbation in cavities, etc.) provide accurate results for homogeneous materials in the microwave range. These methods are based on the measurement of the reflection and the transmission coefficients of a sample [1], [2]. This paper focuses on guided space techniques.

The heterogeneous materials are composed of a host material containing various inclusions. To consider this mixing as homogeneous, the dimensions of the inclusions must be small compared to the wavelength and the dimensions of the sample. The concentration of these inclusions must also be relatively low. Moreover, the sample must be representative of the material. In these conditions, an effective permittivity ϵ_{eff} and an effective permeability μ_{eff} can be defined, and used for further computations.

Since the end of the last century, many theoretical models have been developed, in particular the effective medium theories (EMT) [3]. Unfortunately, the range of validity of these theories is very narrow and the geometrical and electromagnetic parameters of the material components are to be known exactly. Even if these assumptions are fullfilled, the use of these formulas remains exceptional since several simplifications are necessary to perform calculations and the resulting conditions for applying these theories are very restrictive. Furthermore, it is impossible to control perfectly the manufacturing of a material and to know the exact geometrical parameters. Moreover, these theories are valid only for particular materials. So it is necessary to be able to perform measurements.

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Classic methods employed for measuring the properties of homogeneous materials are based on the propagation of only the dominant mode, are not suitable for heterogeneous materials. Indeed, the experience proves that higher-order modes may be excited when measuring heterogeneous materials.

This paper presents a new formulation for applying the measured scattering parameters S_{ij} of a sample composite material placed in a transmission line. It is a macroscopic model taking into account the excitation of higher-order modes inside the sample.

In the first part of the paper, the encountered problems are exhibited.

It is followed by an analysis of the modes able to resonate in the sample. We show how the resonances of higher-order modes are defined.

We next discuss the perturbed line model. Each higher-order mode is presented as an RLC resonator. The characteristic parameters of the resonators, (R, L, C) or (ω_0, Q_0, β), are derived from the measured scattering parameters. It is shown how the intrinsic properties of materials are determined from the measurements.

Finally, this method is applied to determine the parameters ϵ_{eff} and μ_{eff} of various composite materials. Our results agree well with measurements obtained by other techniques.

II. Presentation of the Problem

Although the studied materials are heterogeneous, fluctuations may occur on measured curves [4]. Obviously they are not predictable from effective medium theory. These resonances are due to the excitation of higher-order modes in the sample [5], caused by the scattering of the incident wave from the heterogenities in the sample. These modes are not taken into account in classic computations.

Resonances are shown in two examples. In the first, Fig. 1, we show the results for the two samples of a same material, that verifies the homogenization conditions, but which are measured in two different coaxial lines: one in APC7 (outer diameter 7 mm, and inner diameter 3.04 mm), and one in APL50 (external diameter 50 mm, and internal diameter 21,71 mm). Results for ϵ' and ϵ'' (real and imaginary parts of $\epsilon_r = \epsilon' - j\epsilon''$) are different, as shown in Fig. 1.

In the second example, two samples of a same material, of different thicknesses are measured (Fig. 2). The values of ϵ' and ϵ'' are again different.

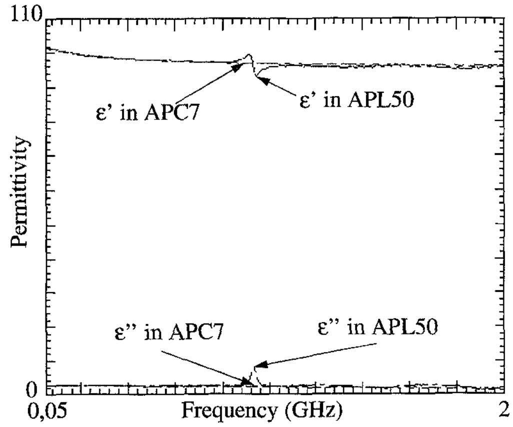

Fig. 1. Measurements of two samples of a same material in two different coaxial lines.

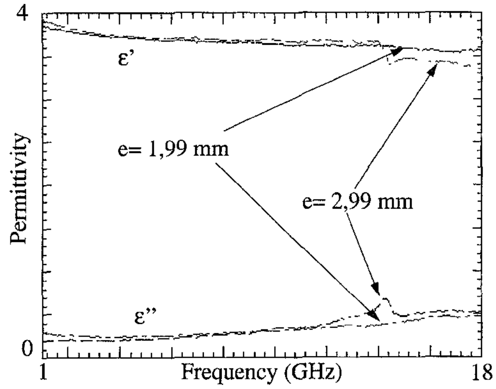

Fig. 2. Measurements of two samples of a same material, of different thickness.

Both examples show that the results depend on the coupling between the sample and the cell, and consequently they should not be used as they stand for the material properties determination.

The observed resonances are induced by the excitation of higher-order modes [6], [7]. Indeed, an essential assumption to apply the classic reflection/transmission computation [8], is that only the dominant mode propagates inside the measurement cell. Consequently, a new model must be developed.

III. Determination of the Higher-Order Modes

The measurement cells are ordinarily used in the frequency range where only the dominant mode propagates in air. When a sample having a dielectric constant greater than one is inserted, other modes can propagate inside the sample. These higher-order modes are, however, not excited in homogeneous materials, contrary to heterogeneous ones.

We note that the observed resonance frequencies are not in agreement with cut-off frequencies of a medium-filled cell. To explain the observed frequency shifts, we have constructed a model which takes into account the finite size of the sample material. Let us consider the sample as dielectric resonator of thickness h. The measurement cell is either a waveguide or a coaxial line (Fig. 3).

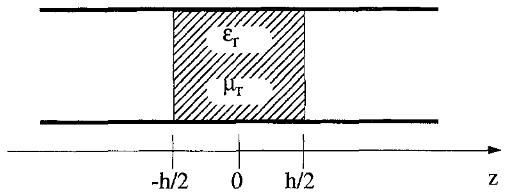

Fig. 3. A sample in a measurement cell is considered as a dielectric resonator.

We assume that the sample is homogenizable and that the measurement cell has perfectly conducting walls. In addition, a third index, p, is introduced to define resonant modes, henceworth modes are denoted by TE_{mnp} and TM_{mnp}. The longitudinal index p corresponds to the periodicity in units of h along the z-axis (p is not an integer since the interfaces sample-air are not perfect magnetic walls [9]).

The device presented in Fig. 3 is symmetrical with regard to the plane defined by $z = 0$. Therefore, the field can be expanded in even and odd modes. Even modes are obtained by substituting the symmetrical plane by an electric wall while odd modes are obtained by substituting the symmetrical plane by a magnetic wall.

When expressing a stationary wave in the region $-h/2 < z < h/2$ subject to the continuity relations of the fields in the planes $z = h/2$, and $z = h/2$, a system of complex equations is obtained for the even and odd modes

$$\begin{cases} \gamma^2 = k^2 - (\epsilon' - j\epsilon'')(\mu' - j\mu'')\epsilon_0\mu_0\omega_0^2 \\ \alpha_0^2 = k^2 - \epsilon_0\mu_0[\mathrm{Re}\,(\omega_0)]^2 \\ -th\left(\frac{\gamma h}{2}\right) = \eta_r & \text{for an even mode} \\ -th\left(\frac{\gamma h}{2}\right) = \frac{1}{\eta_r} & \text{for an odd mode} \end{cases} \quad (1)$$

$$\text{for a TE mode:} \quad \eta_r = \frac{(\mu' - j\mu'')\alpha_0}{\gamma} \quad (2)$$

$$\text{for a TM mode:} \quad \eta_r = \frac{\gamma}{(\epsilon' - j\epsilon'')\alpha_0}. \quad (3)$$

In these formulas, k^2 is the eigen wavenumber of the mode and may involve the effect of metallic losses. α_0 is the attenuation constant in air, ω_0 is the pulsation and γ is the propagation constant in the sample

$$\gamma = \alpha + j\beta \quad (4)$$

where α is the attenuation constant and β is the phase constant of the wave in the sample.

The unknowns are α_0, which is real, and ω_0 and γ which are complex. Their solutions are obtained numerically.

In order to define the parameters characterizing the perturbation, we transform the S-matrix into a normalized impedance matrix, and analyze the imaginary part of the impedance.

Parameters of the resonant curve, as shown in Fig. 4, are:

- Λ Coupling coefficient.
- f_0 Central frequency of the resonance.
- Q_0 Quality factor.

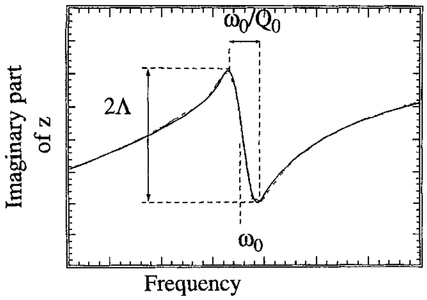

Fig. 4. Parameters characterizing a resonance.

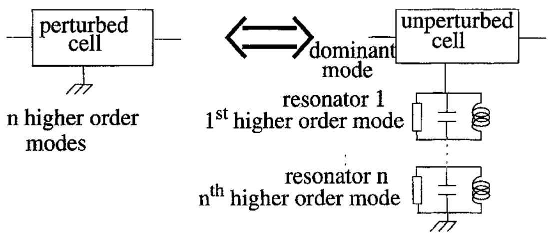

Fig. 5. Model of the perturbed cell.

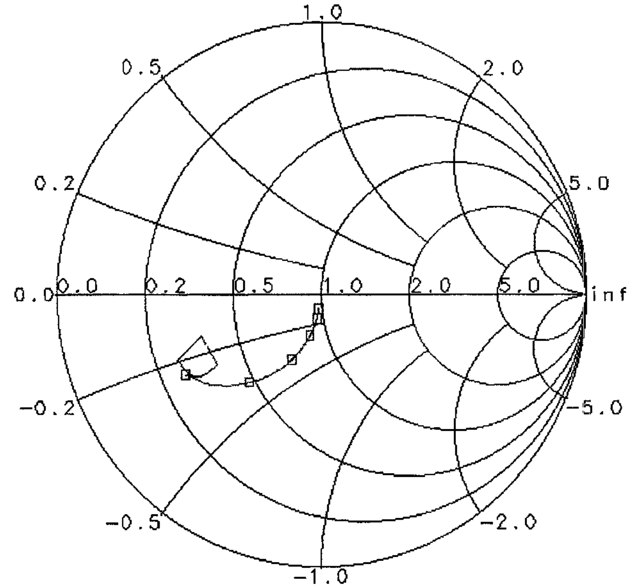

Fig. 6. The observation of the S_{11} parameter justifies the choice of the model.

f_0 and Q_0 can be obtained from α_0, ω_0 and γ

$$f_0 = \frac{\mathrm{Re}\,(\omega_0)}{2\pi} \tag{5}$$

$$Q_0 = \frac{\mathrm{Re}\,(\omega_0)}{2\,\mathrm{Im}\,(\omega_0)}. \tag{6}$$

Moreover, index p is given by

$$p = \frac{\beta h}{\pi}. \tag{7}$$

In this way, we are thus able to determine the possible resonant mode in the sample. The coupling coefficient, Λ, however, is inaccessible since it depends on the microscopic structure of the material while our knowledge of the material is only macroscopic.

IV. Modeling by Resonators

Our aim is to find a method of determining the effective permittivity and permeability of a heterogeneous material from microwave measurements performed in a waveguide or coaxial line. The suggested model is able to account for the effects of higher-order modes by representing them in form of RLC resonators.

In the particular case where the transmission line is excited in a frequency range where only the dominant mode propagates, no higher-order mode is generated in a homogeneous sample. The scattering parameters of this two port can be analytically computed if the thickness and the properties of the sample material are known. Henceforward, let us call such system an unperturbed cell. The perturbed cell is equivalent to an unperturbed cell in series with resonators. The model under consideration is shown in Fig. 5.

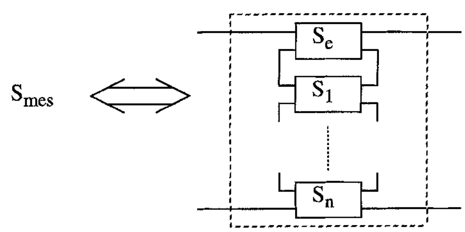

Fig. 7. Use of scattering matrices to perform calculations. S_e: scattering matrix of the sample if only the dominant mode propagates. S_{mes}: scattering parameters obtained by the measurement. $S_1, \cdots, S_n$: scattering matrices corresponding to each resonator.

A. Justification of the Model

First, excited higher-order modes lose almost all their energy in metallic losses in the walls of the cell because they are evanescent outside the sample. Therefore, the resonators must be inserted between an unperturbed line and the ground.

Second, the R, L, C elements must be connected in parallel. Otherwise a lack of energy could be observed without resonance of higher-order modes (for instance with L and C equal to zero, and R not equal to zero). Whereas the observed lack of energy when a higher-order mode resonates, is due to metallic losses in the walls and inclusions. The observation of measured S_{11} on a Smith chart confirms this assumption (Fig. 6).

Finally, each higher-order mode of the perturbed cell is associated with a resonator. The unknowns are the equivalent resistance R, inductance L, capacitance C. They are computed from the resonance frequency, the coupling, and the quality coefficients of the resonator [see (13)–(15)].

The problem can be simplified by use of scattering matrices. S_{mes} is the measured data and $S_1, \cdots, S_n$ are obtained by using (11) and (12). S_e is the unknown matrix from which ϵ_{eff} and μ_{eff} are extracted.

In this step, each S matrix is transformed into a Z matrix using

$$Z = (I + S)(I - S)^{-1} \tag{8}$$

where I is the unity matrix.

As mentioned above, a unique solution exists for each resonator. Width, amplitude, central frequency constitute three independent parameters leading to the determination of R, L, C.

S_e can then be expressed

$$S_e = (Z - I)(Z + I)^{-1} \tag{9}$$

where

$$Z_e = Z_{\mathrm{mes}} - \sum_{i=1}^{n} Z_i. \tag{10}$$

Finally, the classic calculation provides ϵ_{eff} and μ_{eff} from S_e since only the dominant mode is supposed to propagate inside the unperturbed cell.

To perform the above calculation, the S-matrices and Z-matrices of the resonators must be known. The S and Z matrix of a symmetrically diffracting parallel resonator are

$$S = \begin{bmatrix} \frac{-1}{1+2z} & \frac{2z}{1+2z} \\ \frac{2z}{1+2z} & \frac{-1}{1+2z} \end{bmatrix} \quad \text{and} \quad Z = z\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \tag{11}$$

with

$$z = \frac{jRL\omega}{R(1 - LC\omega^2) + jL\omega}. \tag{12}$$

The parameters R, L, C can then be expressed

$$L = \frac{\Lambda}{2\pi f_0 Q_0} \tag{13}$$

$$R = \Lambda \tag{14}$$

$$C = \frac{Q_0}{2\pi f_0 \Lambda}. \tag{15}$$

V. Results

The following procedure is applied for measurements of several homogenizable samples placed in a coaxial line or rectangular waveguide. The parameters R, L, C of the resonators are determined from the measurement. By substracting matrices for resonators, the S-matrix of the cell of the model, where only the dominant mode propagates, can be obtained from which, the parameters ϵ_{eff} and μ_{eff} of the sample can be derived by a classic calculation.

First, numerical results are presented. Table I presents a comparison of the quality factor derived from (6) and (13)–(15) for three materials. Table II presents a comparison of the resonance frequencies derived from (5) and (13)–(15) for the same materials. Also shown are the characteristics of the resonating modes.

Results show a good agreement between measured and calculated values.

One case is studied in detail to prove that obtained results by taking into account the correction with the developed model agree well with the intrinsic characteristics of the materials.

TABLE I
Quality Factor

material No	description	model Eq. (13) to (15)	predicted Eq. (6)	$f_0/\Delta f$ in curves
1	metallic spheres in a dielectric resin	55	48	54
2	mixing of dielectric materials	37	43	42
3	carbon fibers in a dielectric	4,8	8,4	4,6

TABLE II
Resonance Frequencies (GHz) and Resonating Modes

material No	model Eq. (13) to (15)	predicted Eq. (5)	read in curves	mode	p from Eq. (7)
1	16,3	16,3	16,3	TE_{11p}	0,21
2	0,93	0,94	0,93	TE_{11p}	0,18
3	16,6	16,2	16	TE_{21p}	0,36

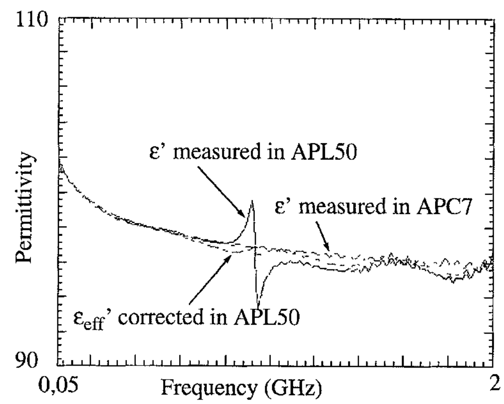

Fig. 8. Comparison of two measurements and a corrected measurement to illustrate performances of the model.

Material No. 2 is tested which is a mixture of dielectric inclusions [with $\epsilon\#\epsilon_0(5000 - j30)$] in epoxy resin [with $\epsilon\#\epsilon_0(3-j0)$]. A sample is measured in a coaxial line of 7 mm in outer diameter (APC7). This measurement is valid because solving (1) for a given permittivity proves that no higher-order modes can be excited below 2,9 GHz. Another sample is measured in a 50 mm diameter coaxial line (APL50). The analytical calculation shows that the first higher-order mode can resonate at 0,93 GHz in the sample. The measurement shows presence of a resonance. The model is used to correct the measurement. The three curves are compared in Fig. 8. We notice that the result obtained with the APC7 measurement and the corrected APL50 measurement are in a good agreement, thus demonstrating that the procedure gives the permittivity of the measured material.

1) Coaxial line APC7: Results for the effective permittivity of the material No. 1 (described in Table I) are shown in Fig. 9. The perturbation is eliminated via the correction, so the model is valid for mixtures of metallic inclusions inside a dielectric matrix.

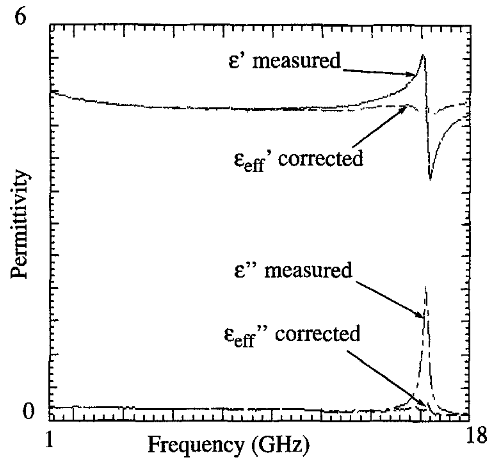

Fig. 9. Use of the model for the material No. 1, composed of metallic spheres in a dielectric resin.

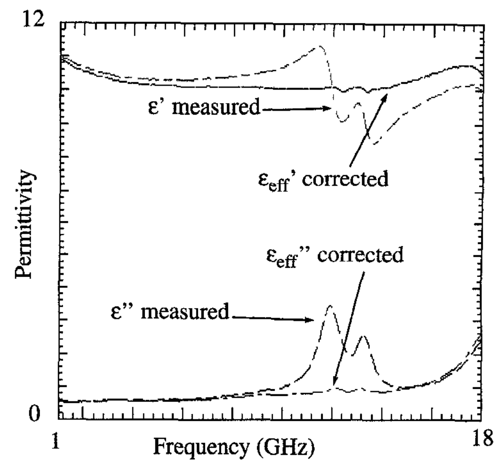

Fig. 11. Use of the model for a measurement showing two higher-order modes.

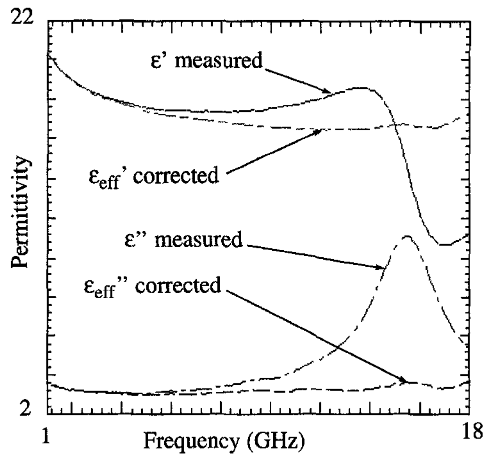

Fig. 10. Use of the model for a measurement showing a large width resonance.

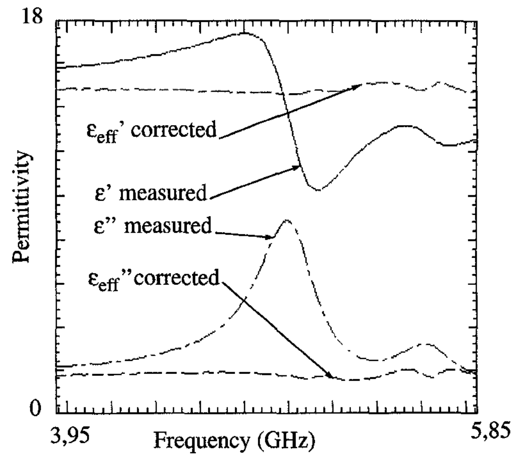

Fig. 12. Use of the model for a measurement performed in a rectangular waveguide.

Results for the material No. 3 are shown in Fig. 10. The corrected data are in a good agreement with predicted values, in spite of the large width of the resonance.

In the previous examples, only one higher-order mode is excited. Now, two resonances appear in Fig. 11. Therefore two RLC resonators should be considered in the correction. The approach is still reliable as we can see in Fig. 11. The sample is composed from carbon fibers and glass particles inside a dielectric resin.

2) Rectangular waveguide: The model takes into account the wave impedance of the cell but the same principle is applied. The impedance, Z_0, of the measurement cell must be introduced in the computation
-for a coaxial line, the characteristic impedance is

$$Z_0 = 50\ \Omega \tag{16}$$

for a rectangular waveguide, the wave impedance is

$$Z_0 = \frac{2b}{a}\sqrt{\frac{\mu_0}{\epsilon_0}}\frac{1}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} \tag{17}$$

where

- **b** Height of the guide.
- **a** Width of the guide.
- $\mathbf{f}_c$ Cut-off frequency of the guide.

In Fig. 12, perturbations due to the resonance of modes TE_{20} and TE_{11} are visible. So, two *RLC* resonators are considered. The cell is a rectangular waveguide of 22.15 mm × 47.55 mm. The sample is composed from carbon fibers inside a dielectric resin.

These various examples show that the developed procedure is reliable for many categories of mixtures and for different measurement cells.

VI. Conclusion

We have developed a new model which enables us to correctly interpret measurements of heterogeneous composite materials performed in guided space. Until now only measurements in free space were usable.

The presented procedure takes into account the higher-order modes resonating inside the sample. The advantage of this model is that it is built directly on physical phenomena appearing in the cell during measurements. In the model, the metallic losses are accounted for via the quality factor. Our method is valid for any kind of waveguide measuring cell, in particular for the coaxial and rectangular waveguides and has proved useful for a number of materials. The characteristics of the material components are not used in our analysis, contrary to the effective media formulas for instance.

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